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# ANALYSIS OF $V_e$-DEGREE AND $E_v$-DEGREE TOPOLOGICAL INDICES OF SILICATE AND OXYGEN NETWORKS

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**Abstract:** Recently, two novel degree based concepts have been defined in graph theory; $E_v$-degrees and $V_e$-degrees. Motivated by chemical applications of topological indices in the QSPR/QSAR analysis, we define $V_e$-degree re-defined versions of Zagreb indices ($V_e - ReZG_1(G)$, $V_e - ReZG_2(G)$, $V_e - ReZG_3(G)$) and $V_e$-degree of SK indices ($V_e - SK(G)$, $V_e - SK_1(G)$, $V_e - SK_2(G)$) as parallel to their corresponding classical degree versions. Further-more, we obtain $V_e$-degree $V_e - ReZG_1(G)$, $V_e - ReZG_2(G)$, $V_e - ReZG_3(G)$, $V_e - SK(G)$, $V_e - SK_1(G)$, $V_e - SK_2(G)$ and $E_v$-degree $E_v - {}^mM(G)$, $E_v - I(G)$, $E_v - F(G)$ of topological indices of some standard class of graphs like, path, cycle, complete, star, wheel and complete bipartite graphs. Also we compute $V_e - ReZG_1(G)$, $V_e - ReZG_2(G)$, $V_e - ReZG_3(G)$, $V_e - SK(G)$, $V_e - SK_1(G)$ and $V_e - SK_2(G)$ topological indices of some silicate oxygen networks such as dominating oxide network ($DOX$), regular triangulate oxide network (RTOX), dominating silicate network (DSL) and derive analytical formulae of these networks. Additionally, we analyze the numerical and graphical comparison of the networks.

**Keywords and Phrases:** SK indices, re-defined Zagreb indices, $V_e$-degree and

$E_v$-degree indices.

**2020 Mathematics Subject Classification:** 05C05, 05C12, 05C35.

## 1. Introduction and Preliminaries

Topological indices are used in the process of correlating the chemical structures with various characteristics such as boiling points and molar heats of formation. Chemical reaction network theory is a branch of applied mathematics aimed at simulating the structure-activity relationship in real-world chemical systems. Since its origin in the nineteenth century, it has grown in popularity among scientists, owing primarily to advances in organic chemistry and theoretical chemistry. Because of the computational architecture, it has also gotten a lot of attention from pure mathematicians. A molecular graph is a simple graph whose vertices correspond to the atoms and whose edges correspond to the bonds.

Let $G = (V, E)$ be a finite, undirected graph without loops and multiple edges with $V$ as vertex set and $E$ as edge set. Let $|V| = n$ and $|E| = m$. For a graph $G$, a vertex $v$, $deg(v)$ show the number of edges that incident to $v$. The set of all vertices which adjacent to $v$ is called the open neighborhood of $v$ and denoted by $N(v)$. If we add the vertex $v$ to $N(v)$, then we get the closed neighborhood of $v$, $N[v]$. For unexplained graph terminology and notation refer [8, 9].

The concept of re-defined Zagreb ($ReZG$) indices was recently introduced by Ranjini et al. is in [11], which are defined as follows.

The first re-defined Zagreb index of a graph $G$ [11], is defined as

$$ReZG_1(G) = \sum_{uv \in E(G)} \frac{deg(u) + deg(v)}{deg(u)deg(v)}.$$

The second re-defined Zagreb index of a graph $G$ [11], is defined as

$$ReZG_2(G) = \sum_{uv \in E(G)} \frac{deg(u)deg(v)}{deg(u) + deg(v)}.$$

The third re-defined Zagreb index of a graph $G$ [11], is defined as

$$ReZG_3(G) = \sum_{uv \in E(G)} (deg(u)deg(v))(deg(u) + deg(v)).$$

The concept of SK-index ($SK$) was recently introduced by Shigehalli et al. is in [14], which are defined as follows.

The SK index of a graph $G$ [14], is defined as

$$SK(G) = \sum_{uv \in E(G)} \frac{deg(u) + deg(v)}{2}.$$

The $SK_1$ index of a graph $G$ [14], is defined as

$$SK_1(G) = \sum_{uv \in E(G)} \frac{deg(u)deg(v)}{2}.$$

The $SK_2$ index of a graph $G$ [14], is defined as

$$SK_2(G) = \sum_{uv \in E(G)} \left( \frac{deg(u) + deg(v)}{2} \right)^2.$$

Recently two novel degree concepts '$E_v$-degree' and '$V_e$-degree' for the vertices and edges have been defined in Chellali et al [3]. The definition of $V_e$-degree defined as below.

**Definition 1.1.** [6] *Let $G$ be a connected graph and $v \in V(G)$. The $V_e$-degree of the vertex $v$, $deg_{ve}v$, equals the number of different edges that incident to any vertex from the closed neighbourhood of $v$.*

The $E_v$-degree of an edge is defined as below.

**Definition 1.2.** [6] *Let $G$ be a connected graph and $e = uv \in E(G)$. The $E_v$-degree of the edge $e$, $deg_{ev}e$, equals the number of vertices of the union of the closed neighborhoods of $u$ and $v$.*

The authors [3] have found that the total $E_v$-degree and $V_e$-degree for any graph are closely related to well-known degree based topological index: the first Zagreb index. Because of this result and Motivated by the recent results on $V_e$-degree indices [6], we now define $V_e$-degree re-defined versions of Zagreb indices ($V_e - ReZG_1(G)$, $V_e - ReZG_2(G)$, $V_e - ReZG_3(G)$) and $V_e$-degree of SK indices ($V_e - SK(G)$, $V_e - SK_1(G)$, $V_e - SK_2(G)$) indices of the molecular graph as follows.

The first $V_e$ degree re-defined Zagreb index of a graph $G$ define as

$$ReZG_1^{ve}(G) = \sum_{uv \in E(G)} \frac{deg_{ve}(u) + deg_{ve}(v)}{deg_{ve}(u)deg_{ve}(v)}.$$

The second $V_e$ degree re-defined Zagreb index of a graph $G$ define as

$$ReZG_2^{ve}(G) = \sum_{uv \in E(G)} \frac{deg_{ve}(u)deg_{ve}(v)}{deg_{ve}(u) + deg_{ve}(v)}.$$

The third $V_e$ degree re-defined Zagreb index of a graph $G$ define as

$$ReZG_3^{ve}(G) = \sum_{uv \in E(G)} (deg_{ve}(u)deg_{ve}(v))(deg_{ve}(u) + deg_{ve}(v)).$$

The $V_e$ degree SK index of a graph $G$ define as

$$SK^{ve}(G) = \sum_{uv \in E(G)} \frac{deg_{ve}(u) + deg_{ve}(v)}{2}.$$

The $V_e$ degree $SK_1$ index of a graph $G$ define as

$$SK_1^{ve}(G) = \sum_{uv \in E(G)} \frac{deg_{ve}(u)deg_{ve}(v)}{2}.$$

The $V_e$ degree $SK_2$ index of a graph $G$ define as

$$SK_2^{ve}(G) = \sum_{uv \in E(G)} \left(\frac{deg_{ve}(u) + deg_{ve}(v)}{2}\right)^2.$$

The modified $E_v$-degree Zagreb index of a graph $G$ [10], defined as

$${}^{m}M^{ev}(G) = \sum_{e \in E(G)} \frac{1}{deg_{ev}(e)^2}.$$

The $E_v$-degree inverse index of a graph $G$ [10], defined as

$$I^{ev}(G) = \sum_{e \in E(G)} \frac{1}{deg_{ev}(e)}.$$

The $E_v$-degree $F$-index of a graph $G$ [10], defined as

$$F^{ev}(G) = \sum_{e \in E(G)} deg_{ev}(e)^3.$$

For more on $V_e$-degree and $E_v$-degree topological indices of molecular graphs which are in [1-5, 7, 10, 12].

The present paper is organized as follows: In section 2, we obtain $V_e$-degree and $E_v$-degree topological indices for some standard class of graphs like path, cycle, complete, star, wheel and complete bipartite graphs. In section 3, 4 and 5, we derive

$V_e$-degree indices for the dominating oxide networks ($DOX(n)$), regular triangulate oxide networks ($RTOX(n)$) and dominating silicate networks ($DSL(n)$). In section 6, we study the numerical and graphical comparison of $DOX(n)$, $RTOX(n)$ and $DSL(n)$ networks.

## 2. $V_e$-degree and $E_v$-degree topological indices for some standard class of graphs

**Proposition 2.1.** *For path graph $P_n$ is*

1. $ReZG_1^{ve}(P_n) = \begin{cases} 2 & if\ n = 2, 3, \\ \frac{7}{3} & if\ n = 4, \\ \frac{3n+2}{6} & if\ n \geq 5. \end{cases}$

2. $ReZG_2^{ve}(P_n) = \begin{cases} \frac{1}{2} & if\ n = 2, \\ 2 & if\ n = 3, \\ \frac{39}{10} & if\ n = 4, \\ \frac{70n-146}{35} & if\ n \geq 5. \end{cases}$

3. $ReZG_3^{ve}(P_n) = \begin{cases} 2 & if\ n = 2, \\ 32 & if\ n = 3, \\ 114 & if\ n = 4, \\ 128n - 412 & if\ n \geq 5. \end{cases}$

4. $SK^{ve}(P_n) = \begin{cases} 1 & if\ n = 2, \\ 4n - 8 & if\ n \geq 3. \end{cases}$

5. $SK_1^{ve}(P_n) = \begin{cases} \frac{1}{2} & if\ n = 2, \\ 4 & if\ n = 3, \\ \frac{21}{2} & if\ n = 4, \\ 8n - 22 & if\ n \geq 5. \end{cases}$

6. $SK_2^{ve}(P_n) = \begin{cases} 1 & if\ n = 2, \\ 8 & if\ n = 3, \\ \frac{43}{2} & if\ n = 4, \\ 16n - 43 & if\ n \geq 5. \end{cases}$

7. ${}^{m}M^{ev}(P_n) = \begin{cases} \frac{1}{4} & if\ n = 2, \\ \frac{9n+5}{144} & if\ n \geq 3. \end{cases}$

8. $I^{ev}(P_n) = \begin{cases} \frac{1}{2} & if\ n = 2, \\ \frac{3n-1}{12} & if\ n \geq 3. \end{cases}$

9. $F^{ev}(P_n) = \begin{cases} 8 & if\ n = 2, \\ 64n - 138 & if\ n \geq 3. \end{cases}$

**Proposition 2.2.** *For cycle graph $C_n$ is*

1. $ReZG_1^{ve}(C_n) = \begin{cases} \frac{4}{3} & if\ n = 3, \\ \frac{n}{2} & if\ n \geq 4. \end{cases}$

2. $ReZG_2^{ve}(C_n) = \begin{cases} 3 & if\ n = 3, \\ 2n & if\ n \geq 4. \end{cases}$

3. $ReZG_3^{ve}(C_n) = \begin{cases} 108 & if\ n = 3, \\ 128n & if\ n \geq 4. \end{cases}$

4. $SK^{ve}(C_n) = \begin{cases} 6 & if\ n = 3, \\ 4n & if\ n \geq 4. \end{cases}$

5. $SK_1^{ve}(C_n) = \begin{cases} 9 & if\ n = 3, \\ 8n & if\ n \geq 4. \end{cases}$

6. $SK_2^{ve}(C_n) = \begin{cases} 36 & if\ n = 3, \\ 16n & if\ n \geq 4. \end{cases}$

7. ${}^{m}M^{ev}(C_n) = \begin{cases} \frac{1}{3} & if\ n = 3, \\ \frac{n}{16} & if\ n \geq 4. \end{cases}$

8. $I^{ev}(C_n) = \begin{cases} 1 & if\ n = 3, \\ \frac{n}{4} & if\ n \geq 4. \end{cases}$

9. $F^{ev}(C_n) = \begin{cases} 81 & if\ n = 3, \\ 64n & if\ n \geq 4. \end{cases}$

**Proposition 2.3.** *For complete graph $K_n$ $(n \geq 3)$ is*

1. $ReZG_1^{ve}(K_n) = 2,$

2. $ReZG_2^{ve}(K_n) = \frac{n^2(n-1)^2}{8},$

3. $ReZG_3^{ve}(K_n) = \frac{n^4(n-1)^4}{8},$

4. $SK^{ve}(K_n) = \frac{n^2(n-1)^2}{4},$

5. $SK_1^{ve}(K_n) = \frac{n^3(n-1)^3}{16},$

6. $SK_2^{ve}(K_n) = \frac{n^3(n-1)^3}{8}$,
7. ${}^{m}M^{ev}(K_n) = \frac{n-1}{2n}$,
8. $I^{ev}(K_n) = \frac{n-1}{2}$,
9. $F^{ev}(K_n) = \frac{n^4(n-1)}{2}$.

**Proposition 2.4.** *For star graph $K_{1,n}$ $(n \geq 3)$ is*

1. $ReZG_1^{ve}(K_{1,n}) = 2$,
2. $ReZG_2^{ve}(K_{1,n}) = \frac{n^2}{2}$,
3. $ReZG_3^{ve}(K_{1,n}) = 2n^4$,
4. $SK^{ve}(K_{1,n}) = n^2$,
5. $SK_1^{ve}(K_{1,n}) = \frac{n^3}{2}$,
6. $SK_2^{ve}(K_{1,n}) = n^3$,
7. ${}^{m}M^{ev}(K_{1,n}) = \frac{n}{(n+1)^2}$,
8. $I^{ev}(K_{1,n}) = \frac{n}{n+1}$,
9. $F^{ev}(K_{1,n}) = n(n+1)^3$.

**Proposition 2.5.** *For wheel graph $W_{1,n}$, $(n \geq 4)$ is*

1. $ReZG_1^{ve}(W_{1,n}) = \frac{7n+4}{2n+8}$,
2. $ReZG_2^{ve}(W_{1,n}) = \frac{7n^3+32n^2+16n}{6n+8}$,
3. $ReZG_3^{ve}(W_{1,n}) = 8n^4 + 56n^3 + 128n^2 + 128n$,
4. $SK^{ve}(W_{1,n}) = \frac{5n^2+12n}{2}$,
5. $SK_1^{ve}(W_{1,n}) = \frac{3n^3+16n^2+16n}{2}$,
6. $SK_2^{ve}(W_{1,n}) = \frac{13n^3+56n^2+80n}{4}$,
7. ${}^{m}M^{ev}(W_{1,n}) = \frac{n^2+(n+1)^2}{n(n+1)^2}$,

8. $I^{ev}(W_{1,n}) = \frac{2n+1}{n+1}$,

9. $F^{ev}(W_{1,n}) = n[n^3 + (n+1)^3]$.

**Proposition 2.6.** *For complete bipartite graph $K_{m,n}$, $(m, n \geq 2)$ is*

1. $ReZG_1^{ve}(K_{m,n}) = 2$,

2. $ReZG_2^{ve}(K_{m,n}) = \frac{(mn)^2}{2}$,

3. $ReZG_3^{ve}(K_{m,n}) = 2(mn)^4$,

4. $SK^{ve}(K_{m,n}) = (mn)^2$,

5. $SK_1^{ve}(K_{m,n}) = \frac{(mn)^3}{2}$,

6. $SK_2^{ve}(K_{m,n}) = (mn)^3$,

7. ${}^{m}M^{ev}(K_{m,n}) = \frac{mn}{(m+n)^2}$,

8. $I^{ev}(K_{m,n}) = \frac{mn}{m+n}$,

9. $F^{ev}(K_{m,n}) = mn(m+n)^3$.

## 3. $V_e$-degree indices for Dominating Oxide Networks $DOX(n)$

In this section, we consider the graph of a dominating oxide network $DOX(n)$, see Fig. 1.

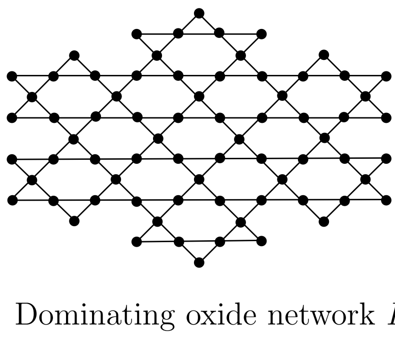

Figure 1: Dominating oxide network $DOX(n)$.

Let $G$ be the graph of $DOX(n)$. The graph dominating oxide network $DOX(n)$ has $54n^2 - 54n + 18$ edges. Also there are two types of edges in $G$ based on the degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 4\}, \quad |E_1| = 24n - 12,$
$E_2 = \{uv \in E(G) | d_G(u) = d_G(v) = 4\}, \quad |E_2| = 54n^2 - 78n + 30.$

The partition of the edges based on $V_e$-degree of end vertices of dominating oxide networks is given in Table 1.

Table 1: The $V_e$-degree of the end vertices of edges for dominating oxide networks $DOX(n)$.

| $(deg_{ve}(u), deg_{ve}(v))$ | (7, 10) | (7, 12) | (10, 10) | (10, 12) | (12, 14) | (14, 14) |
|---|---|---|---|---|---|---|
| Number of edges | $12n$ | $12n-12$ | 6 | $12n-12$ | $24n-24$ | $54n^2-114n+60$ |

**Theorem 3.1.** *The $V_e$-degree first re-defined Zagreb ($V_e - ReZG_1$) index of dominating oxide network $DOX(n)$ is*

$$ReZG_1^{ve}(DOX(n)) = \frac{54n^2}{7} - \frac{188n}{35} + \frac{62}{35}.$$

**Proof.** By using the definition of ($V_e - ReZG_1$) index and Table 1, we obtain

$$\begin{aligned} ReZG_1^{ve}(DOX(n)) &= \sum_{uv \in E(G)} \frac{deg_{ve}(u) + deg_{ve}(v)}{deg_{ve}(u) deg_{ve}(v)} \\ &= 12n\left(\frac{7+10}{7\times 10}\right) + 12n - 12\left(\frac{7+12}{7\times 12}\right) + 6\left(\frac{10+10}{10\times 10}\right) \\ &+ 12n - 12\left(\frac{10+12}{10\times 12}\right) + 24n - 24\left(\frac{12+14}{12\times 14}\right) \\ &+ 54n^2 - 114n + 60\left(\frac{14+14}{14\times 14}\right) \end{aligned}$$

After some simplification, we get the desired result.

**Theorem 3.2.** *The $V_e$-degree second re-defined Zagreb ($V_e - ReZG_2$) index of dominating oxide network $DOX(n)$ is*

$$ReZG_2^{ve}(DOX(n)) = 378n^2 + \left(\frac{720}{11} + \frac{2016}{13} + \frac{840}{17} + \frac{1008}{19} - 798\right)n - \frac{720}{11} - \frac{2016}{13} - \frac{1008}{19} + 450.$$

**Theorem 3.3.** *The $V_e$-degree third re-defined Zagreb ($V_e - ReZG_3$) index of dominating oxide network $DOX(n)$ is*

$$ReZG_3^{ve}(DOX(n)) = 296352n^2 - 455688n + 185616.$$

**Theorem 3.4.** *The $V_e$-degree $(V_e - SK)$ index of dominating oxide network $DOX(n)$ is*

$$SK^{ve}(DOX(n)) = 756n^2 - 936n + 342.$$

**Theorem 3.5.** *The $V_e$-degree $(V_e - SK_1)$ index of dominating oxide network $DOX(n)$ is*

$$SK_1^{ve}(DOX(n)) = 5292n^2 - 7512n + 2940.$$

**Theorem 3.6.** *The $V_e$-degree $(V_e - SK_2)$ index of dominating oxide network $DOX(n)$ is*

$$SK_2^{ve}(DOX(n)) = 10584n^2 - 14886n + 5769.$$

## 4. $V_e$-degree indices for Regular Triangulate Oxide Networks $RTOX(n)$

In this section, we consider a family of regular triangular oxide networks which is denoted by $RTOX(n), n \geq 3$. The graph $RTOX(5)$, is shown in Fig. 2.

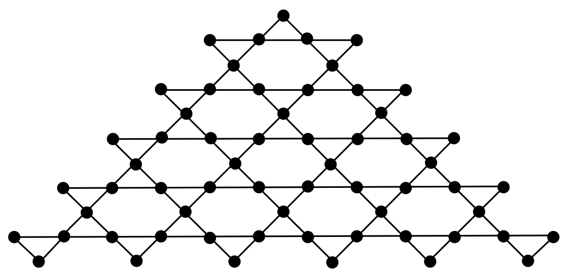

Figure 2: Regular triangular oxide network, $RTOX(5)$.

Let $G$ be the graph of $RTOX(n)$. The graph regular triangular oxide network $RTOX(n)$ has $3n^2 + 6n$ edges. Also there are three types of edges in $G$ based on the degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 2\}$, $|E_1| = 2$,
$E_2 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 4\}$, $|E_2| = 6n$,
$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 4\}$, $|E_3| = 3n^2 - 2$.

The partition of the edges based on $V_e$-degree of end vertices of regular triangular oxide network is given in Table 2.

Table 2: The $V_e$-degree of the end vertices of edges for regular triangular oxide network $RTOX(n)$.

| $(deg_{ve}(u), deg_{ve}(v))\backslash uv \in E(G)$ | Number of edges |
|---|---|
| $(5,5)$ | $2$ |
| $(5,10)$ | $4$ |
| $(7,10)$ | $4$ |
| $(7,12)$ | $6n-8$ |
| $(10,10)$ | $1$ |
| $(10,12)$ | $6$ |
| $(12,12)$ | $6n-9$ |
| $(12,14)$ | $6n-12$ |
| $(14,14)$ | $3n^2-12n+12$ |

**Theorem 4.1.** *The $V_e$-degree first re-defined Zagreb $(V_e - ReZG_1)$ index of regular triangular oxide networks $RTOX(n)$ is*

$$ReZG_1^{ve}(RTOX(n)) = \frac{3n^2}{7} + \frac{48n}{21} - \frac{5n}{7} - \frac{1}{2} + \frac{11}{10} + \frac{12}{7} + \frac{34}{35} - \frac{77}{21}.$$

**Proof.** By using the definition of $(V_e - ReZG_1)$ index and Table 2, we obtain

$$\begin{aligned} ReZG_1^{ve}(RTOX(n)) &= \sum_{uv \in E(G)} \frac{deg_{ve}(u) + deg_{ve}(v)}{deg_{ve}(u)deg_{ve}(v)} \\ &= 2\left(\frac{5+5}{5\times 5}\right) + 4\left(\frac{5+10}{5\times 10}\right) + 4\left(\frac{7+10}{7\times 10}\right) \\ &+ 6n-8\left(\frac{7+12}{7\times 12}\right) + 1\left(\frac{10+10}{10\times 10}\right) + 6\left(\frac{10+12}{10\times 12}\right) \\ &+ 6n-9\left(\frac{12+12}{12\times 12}\right) + 6n-12\left(\frac{12+14}{12\times 14}\right) \\ &+ 3n^2-12n+12\left(\frac{14+14}{14\times 14}\right) \end{aligned}$$

After some simplification, we get the desired result.

**Theorem 4.2.** *The $V_e$-degree second re-defined Zagreb $(V_e - ReZG_2)$ index of regular triangular oxide networks $RTOX(n)$ is*

$$ReZG_2^{ve}(RTOX(n)) = 21n^2 + \left(\frac{504}{13} + \frac{504}{19} - 48\right)n + \frac{40}{3} + \frac{360}{11} - \frac{1008}{13} + \frac{280}{17} - \frac{672}{19} + 40.$$

**Theorem 4.3.** *The $V_e$-degree third re-defined Zagreb ($V_e - ReZG_3$) index of regular triangular oxide networks $RTOX(n)$ is*

$$ReZG_3^{ve}(RTOX(n)) = 16464n^2 - 9336n - 4332.$$

**Theorem 4.4.** *The $V_e$-degree ($V_e - SK$) index of regular triangular oxide networks $RTOX(n)$ is*

$$SK^{ve}(RTOX(n)) = 42n^2 + 39n - 22.$$

**Theorem 4.5.** *The $V_e$-degree ($V_e - SK_1$) index of regular triangular oxide networks $RTOX(n)$ is*

$$SK_1^{ve}(RTOX(n)) = 294n^2 + 12n - 160.5.$$

**Theorem 4.6.** *The $V_e$-degree ($V_e - SK_2$) index of regular triangular oxide networks $RTOX(n)$ is*

$$SK_2^{ve}(RTOX(n)) = 588n^2 + 67.5n - 304.$$

## 5. $V_e$-degree indices for dominating silicate network $DSL(n)$

In this section, we consider dominating silicate network $DSL(n)$. The molecular structure of $DSL(2)$ is shown in Fig. 3.

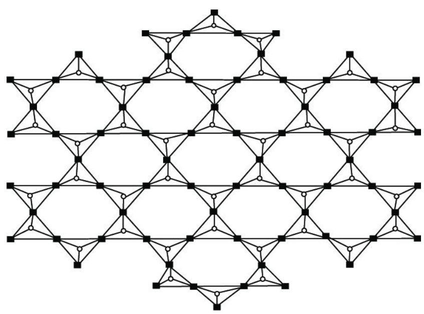

Figure 3: Dominating silicate network $DSL(2)$.

Let $G$ be the dominating silicate network $DSL(n)$. The graph dominating silicate network $DSL(n)$ has $45n^2 - 39n + 12$ vertices and $108n^2 - 108n + 36$ edges. Also there are four types of edges in $G$ based on the degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 3\}$, $|E_1| = 12n - 6$,
$E_2 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 6\}$, $|E_2| = 24n - 12$,
$E_3 = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}$, $|E_3| = 54n^2 - 66n + 24$,
$E_4 = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}$, $|E_3| = 54n^2 - 78n + 30$.

Table 3: The $V_e$-degree of the end vertices of edges for dominating silicate network $DSL(n)$.

| $(deg_{ve}(u), deg_{ve}(v))\backslash uv \in E(G)$ | Number of edges |
|---|---|
| $(12, 13)$ | $12n - 6$ |
| $(13, 20)$ | $12n$ |
| $(12, 20)$ | $12n$ |
| $(12, 23)$ | $24n - 24$ |
| $(15, 23)$ | $12n - 12$ |
| $(15, 26)$ | $54n^2 - 102n + 48$ |
| $(20, 20)$ | $6$ |
| $(20, 23)$ | $12n - 2$ |
| $(23, 26)$ | $24n - 24$ |
| $(26, 26)$ | $54n^2 - 114n + 60$ |

The partition of the edges based on $V_e$-degree of end vertices of dominating silicate network $DSL(n)$ is given in Table 3.

**Theorem 5.1.** *The $V_e$-degree first re-defined Zagreb ($V_e - ReZG_1$) index of dominating silicate networks $DSL(n)$ is*

$$ReZG_1^{ve}(DSL(n)) = \frac{639n^2}{65} + \left(\frac{24}{15} - \frac{89}{13} + \frac{70}{23} - \frac{598}{65} + \frac{281}{115} + \frac{588}{299}\right)n + \frac{3}{5} + \frac{60}{13} - \frac{70}{23} - \frac{25}{26}$$

$$+ \frac{328}{65} - \frac{152}{115} - \frac{43}{230} - \frac{588}{299}.$$

**Proof.** By using the definition of $(V_e - ReZG_1)$ index and Table 2, we obtain

$$\begin{aligned}
ReZG_1^{ve}(DSL(n)) &= \sum_{uv \in E(G)} \frac{deg_{ve}(u) + deg_{ve}(v)}{deg_{ve}(u)deg_{ve}(v)} \\
&= 12n - 6\left(\frac{12+13}{12 \times 13}\right) + 12n\left(\frac{13+20}{13 \times 20}\right) + 12n\left(\frac{12+20}{12 \times 20}\right) \\
&+ 24n - 24\left(\frac{12+23}{12 \times 23}\right) + 12n - 12\left(\frac{15+23}{15 \times 23}\right) \\
&+ 54n^2 - 102n + 48\left(\frac{15+26}{15 \times 26}\right) + 6\left(\frac{20+20}{20 \times 20}\right)
\end{aligned}$$

$$+ \quad 12n - 2\left(\frac{20+23}{20\times 23}\right) + 24n - 24\left(\frac{23+26}{23\times 26}\right)$$
$$+ \quad 54n^2 - 114n + 60\left(\frac{26+26}{26\times 26}\right)$$

After some simplification, we get the desired result.

**Theorem 5.2.** *The $V_e$-degree second re-defined Zagreb ($V_e - ReZG_2$) index of dominating silicate networks $DSL(n)$ is*

$$ReZG_2^{ve}(DSL(n)) = \frac{49842n^2}{41} + \left(\frac{1040}{11} + \frac{2070}{19} + \frac{1872}{25} + \frac{6624}{35} - \frac{39780}{41} + \frac{5520}{43} + \frac{14352}{49} - 1392\right)n$$

$$-\frac{2070}{19} - \frac{936}{25} - \frac{6624}{35} + \frac{18720}{41} - \frac{920}{43} - \frac{14352}{49} + 840.$$

**Theorem 5.3.** *The $V_e$-degree third re-defined Zagreb ($V_e - ReZG_3$) index of dominating silicate networks $DSL(n)$ is*

$$ReZG_3^{ve}(DSL(n)) = 2761668n^2 - 4066620n + 1817272.$$

**Theorem 5.4.** *The $V_e$-degree ($V_e - SK$) index of dominating silicate networks $DSL(n)$ is*

$$SK^{ve}(DSL(n)) = 2511n^2 - 3021n + 1310.$$

**Theorem 5.5.** *The $V_e$-degree ($V_e - SK_1$) index of dominating silicate networks $DSL(n)$ is*

$$SK_1^{ve}(DSL(n)) = 28782n^2 - 39168n + 17354.$$

**Theorem 5.6.** *The $V_e$-degree ($V_e - SK_2$) index of dominating silicate networks $DSL(n)$ is*

$$SK_2^{ve}(DSL(n)) = 59197.5n^2 + 80080.5n + 35182.$$

## 6. Numerical and Graphical Comparison

In the numerical comparison of $V_e$-degree topological indices first, second and third re-defined Zagreb indices and $SK$ index, $SK_1$ index and $SK_2$ index for $DOX(n)$, $RTOX(n)$ and $DSL(n)$, we computed these indices for different values of $n$. It can be observed that when value of n increases, indices values are also increased as shown in Table 4, 5 and 6. The graphical representation of these topological indices are illustrated in Fig. 4, 5 and 6 for the different values of $n$.

Table 4: Numerical comparison of $ReZG_1^{ve}$, $ReZG_2^{ve}$, $ReZG_3^{ve}$, $SK^{ve}$, $SK_1^{ve}$ and $SK_2^{ve}$ indices of $DOX(n)$.

| $n$ | $ReZG_1^{ve}$ | $ReZG_2^{ve}$ | $ReZG_3^{ve}$ | $SK^{ve}$ | $SK_1^{ve}$ | $SK_2^{ve}$ |
|---|---|---|---|---|---|---|
| 1 | 4.1142 | 1029.4209 | 26280 | 162 | 720 | 1467 |
| 2 | 21.8854 | 2638.4258 | 459648 | 1494 | 9084 | 18333 |
| 3 | 55.085 | 5003.4307 | 1485720 | 4338 | 28032 | 56367 |
| 4 | 103.713 | 8124.4356 | 3104496 | 8694 | 57564 | 115569 |
| 5 | 167.7694 | 12001.4405 | 5315976 | 14562 | 97680 | 195939 |
| 6 | 247.2542 | 16634.4454 | 8120160 | 21942 | 148380 | 297477 |
| 7 | 342.1674 | 22023.4503 | 11517048 | 30834 | 209664 | 420183 |
| 8 | 452.509 | 28168.4552 | 15506640 | 41238 | 281532 | 564057 |
| 9 | 578.279 | 35069.4601 | 20088936 | 53154 | 363984 | 729099 |
| 10 | 719.4774 | 42726.465 | 25263936 | 66582 | 457020 | 915309 |

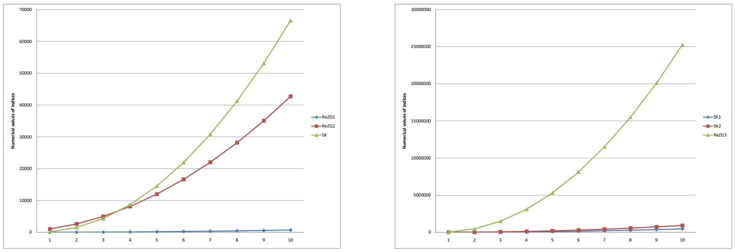


Figure 4: Graphical comparison of $ReZG_1^{ve}$, $ReZG_2^{ve}$, $SK^{ve}$, $SK_1^{ve}$, $SK_2^{ve}$ and $ReZG_3^{ve}$.

Table 5: Numerical comparison of $ReZG_1^{ve}$, $ReZG_2^{ve}$, $ReZG_3^{ve}$, $SK^{ve}$, $SK_1^{ve}$ and $SK_2^{ve}$ indices of $RTOX(n)$.

| $n$ | $ReZG_1^{ve}$ | $ReZG_2^{ve}$ | $ReZG_3^{ve}$ | $SK^{ve}$ | $SK_1^{ve}$ | $SK_2^{ve}$ |
|---|---|---|---|---|---|---|
| 1 | 1.619 | 48.6713 | 21468 | 59 | 145.5 | 351.5 |
| 2 | 4.476 | 128.9668 | 80196 | 224 | 1039.5 | 2183 |
| 3 | 8.19 | 251.2623 | 171852 | 473 | 2521.5 | 5190.5 |
| 4 | 12.761 | 415.5578 | 296436 | 806 | 4591.5 | 9374 |
| 5 | 18.189 | 621.8533 | 453948 | 1223 | 7249.5 | 14733.5 |
| 6 | 24.474 | 870.1488 | 644388 | 1724 | 10495.5 | 21269 |
| 7 | 31.616 | 1160.4443 | 867756 | 2309 | 14329.5 | 28980.5 |
| 8 | 39.615 | 1492.7398 | 1124052 | 2978 | 18751.5 | 37868 |
| 9 | 48.471 | 1867.0353 | 1413276 | 3731 | 23761.5 | 47931.5 |
| 10 | 58.184 | 2283.3308 | 1735428 | 4568 | 29359.5 | 59171 |

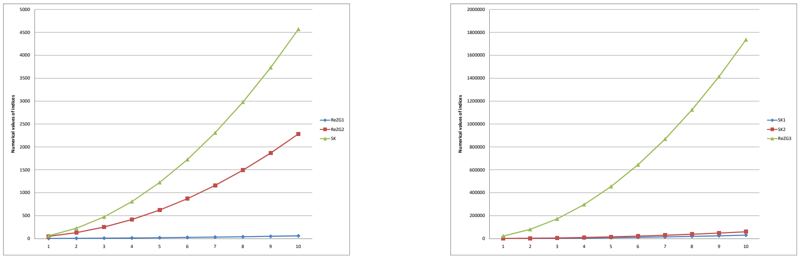


Figure 5: Graphical comparison of $ReZG_1^{ve}$, $ReZG_2^{ve}$, $SK^{ve}$, $SK_1^{ve}$, $SK_2^{ve}$ and $ReZG_3^{ve}$.

Table 6: Numerical comparison of $ReZG_1^{ve}$, $ReZG_2^{ve}$, $ReZG_3^{ve}$, $SK^{ve}$, $SK_1^{ve}$ and $SK_2^{ve}$ indices of $DSL(n)$.

| $n$ | $ReZG_1^{ve}$ | $ReZG_2^{ve}$ | $ReZG_3^{ve}$ | $SK^{ve}$ | $SK_1^{ve}$ | $SK_2^{ve}$ |
|---|---|---|---|---|---|---|
| 1 | 5.6193 | 388.962 | 512320 | 800 | 6968 | 174460 |
| 2 | 28.1186 | 2562.5933 | 4730704 | 5312 | 54146 | 432133 |
| 3 | 70.2793 | 7167.5416 | 14472424 | 14846 | 158888 | 808201 |
| 4 | 132.1014 | 14203.8069 | 29737480 | 29402 | 321194 | 1302664 |
| 5 | 213.5849 | 23671.3892 | 50525872 | 48980 | 541064 | 1915522 |
| 6 | 314.7298 | 35570.2885 | 76837600 | 73580 | 818498 | 2646775 |
| 7 | 435.5361 | 49900.5048 | 108672664 | 103202 | 1153496 | 3496423 |
| 8 | 576.0038 | 66662.0381 | 146031064 | 137846 | 1546058 | 4464466 |
| 9 | 736.1329 | 85854.8884 | 188912800 | 177512 | 1996184 | 5550904 |
| 10 | 915.9234 | 107479.0557 | 237317872 | 222200 | 2503874 | 6755737 |

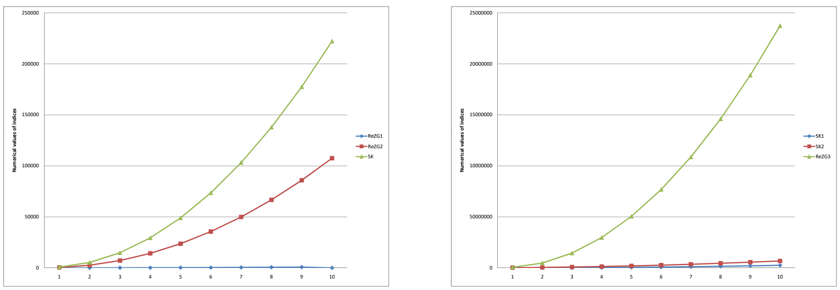


Figure 6: Graphical comparison of $ReZG_1^{ve}$, $ReZG_2^{ve}$, $SK^{ve}$, $SK_1^{ve}$, $SK_2^{ve}$ and $ReZG_3^{ve}$.

## 4. Conclusion

In this study, we define $V_e$-degree re-defined versions of Zagreb indices ($V_e - ReZG_1(G)$, $V_e - ReZG_2(G)$, $V_e - ReZG_3(G)$) and $V_e$-degree of SK indices ($V_e - SK(G)$, $V_e - SK_1(G)$, $V_e - SK_2(G)$). Also we investigated $V_e$-degree topological indices of some silicate oxygen networks such as dominating oxide network ($DOX$), regular triangulate oxide network (RTOX), dominating silicate network (DSL) and derive analytical formulae of these networks. Also we have obtained $V_e$-degree and $E_v$-degree topological indices of some standard class of graphs. One can obtain the $V_e$-degree and $E_v$-degree topological indices of some chemical graphs.

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